MATHEMATICS

Mathematical Measure For Art

With Formula and Geometric Form You May Analyze The Source of Your Delight in Painting and Poetry

By MARJORIE VAN DE WATER

WHY IS IT that men throughout countless ages have taken joy in looking on the moon, a shapely tree, a pearl, or the rhythmical curves of a woman's form?

The secret of this universal aesthetic pleasure in lovely forms lies in mathematics, a Harvard professor of that science, Dr. George D. Birkhoff, tells us. Indeed, he has worked out a mathematical formula from which he can obtain the "aesthetic value" of a shape or form, and this mathematical expression of the beauty of an object conforms to the emotional judgment of those who look upon it.

Not that you need to be a mathematician to appreciate the beautiful. Not at all! It is not necessary to be able to add two and two. Neither is it necessary for you to understand why an object appeals to you as lovely. You needn't say to yourself as you look at the moon, "How symmetrical!" You are perhaps more likely to say with the poet Omar Khayyan, "O Moon of my delight!"

In fact, Dr. Birkhoff says that an intuitive appreciation is better than any attempt to analyze the source of your delight. Nevertheless, that pleasure is due to an unconscious appreciation of the mathematical proportions of the object.

You may not realize it, but the pretty girl is pretty because all her measurements are in the correct relation to each other—if her arms were longer, or her nose shorter, or her height just a little different in relation to her weight, the effect would not be at all the same. Yet you need no tape lines to tell you this. Your ability to perceive these mathematical relations at a glance, without even giving it a deliberate thought, is what gives you your appreciation of all that is beautiful in nature, in art, and in music as well.

It is perhaps no mere coincidence that the word "figure" has a dual meaning.

This finding that beauty can be expressed, if not explained, mathematically has practical importance as well as theoretical interest. The architect need no longer depend upon using forms and designs that have proved their artistic acceptability through generations of aesthetic judgments. He can test his creations against an objective formula as well as against his own "feeling." The designer, the potter, the commercial creator of anything dependent upon its aesthetic qualities for its success can likewise find assistance in this research.

One practical problem on which it has a direct bearing is found in the screen on which motion pictures are projected. What shape should this projected image be? In the days of silent films, the proportion of the length to the height of the screen was 4 to 3. But when sound films were introduced a part of the width of the film was used for the sound track, leaving the picture narrowed. The width of the sound picture is therefore only slightly greater than its height.

Dr. Birkhoff's research has a direct bearing on this very problem.

Good and Bad Movie Frames

The square, he has found, is the most nearly ideal of all straight-line figures. Yet that does not necessarily mean that the square would be the best shape for the motion picture.

"Any obvious numerical ratio of dimensions such as 1 to 1 or 2 to 1 is to be avoided in a picture frame, because it is often desirable that the rectangle be a purely neutral accessory, not producing irrelevent associations," Dr. Birkhoff points out.

The 2 to 1 proportion has an additional disadvantage for a picture frame because it is not well adapted to fill the circular field of effective vision.

But a rectangular form which is nearly a square, but not quite, is positively disagreeable because of the effect of ambiguity, Dr. Birkhoff has found.

The shape of the sound motion picture screen is probably too near to a square to escape this fault of ambiguity. It may seem even nearer than it is because of the fact that the eye overestimates distance up and down as compared with horizontal distances.

Interesting comparisons may be made from the table of aesthetic measures which Dr. Birkhoff has established for various straight-line forms, and which appear in the form of a table in his new book on "Aesthetic Measure," published by the Harvard University Press.

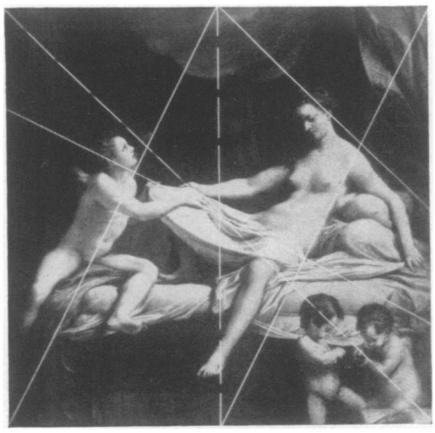
The square heads the list with a value of 1.50. Down among the zeros are a triangle and numerous irregular meaningless forms of odd shapes and proportions.

The swastika, much before the public eye today because it is the chosen symbol of the Nazi, has a value fairly low in the scale—0.33—although its aesthetic value is enhanced, Dr. Birkhoff says, by the fact that despite its notched irregular outline, the form suggests to the mind a regular pattern of horizontal and vertical lines.

A five-pointed star similar to the one appearing on the flag of the United States receives a rating of 0.90, well up near the top of the table.

Dr. Birkhoff, however, does not consider in his calculations the value attached to the symbols because of their associations, but just the aesthetic appeal of their forms. This probably accounts for the fact that although the Greek cross is high in value—0.75 and ninth in order of pleasingness—the Roman cross has a value of only 0.25.

But simple geometric forms are not the only artistic concepts that can be assigned an aesthetic value through the use of Dr. Birkhoff's formulas. He has



GEOMETRY IN ART

Correggio's painting "Danae" is marked by Dr. Birkhoff with white lines to illustrate how the composition involves geometric forms. Although not conspicuous, these forms are suggested by the principal lines of the painting.

included such creations as vases, architectural designs, melodies, and even poetry. Separate scales are, of course, used for the different types of aesthetic creations. The scale of polygons is not the same as the one for poems.

Here is how the formula for poetry looks: $M = \frac{O}{C} = \frac{aa+2r+2m-2ae-2ce}{C}$

The Letter M stands for the aesthetic value of the verse, and the other letters refer to the various artistic qualities such as rhyme, rhythm, alliteration, and so on.

The letter O, which stands for Order, represents the algebraic sum of the values in the numerator of the fraction that follows. The letters aa stand for alliteration and assonance, r for rhyme, n for musical sounds. The two negative values, ae and ce are alliterative excess and excess of consonant sounds, respectively. The letter C in the denominator of the fraction means "complexity."

Counting Beauty

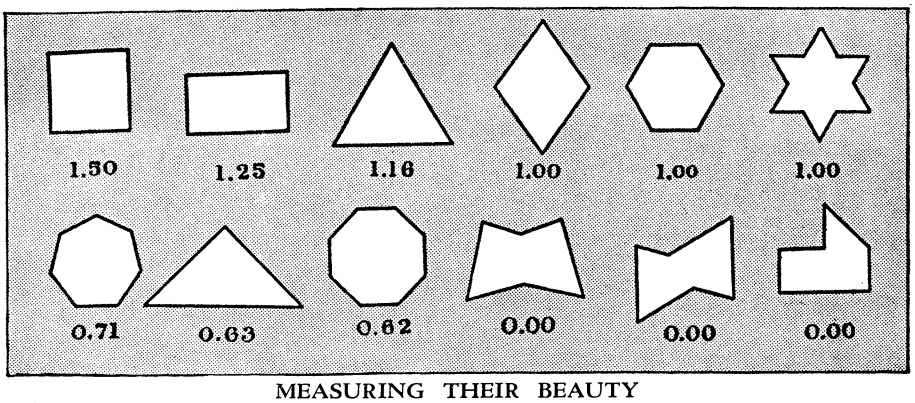
To find the aesthetic value for a poem, Dr. Birkhoff first counts the sounds which are alliterative or similar to other sounds in the same line or the line next preceding. If a sound is like more than two leading sounds in the line or more than four altogether, it is not counted, however, for an excess of alliteration is not pleasing but the reverse. Alliterative sounds are indicated by the capitals in the following line: little Boy Blue, come Blow your horn.

To this total, he adds two more points for each accented vowel sound which rhymes with the same sound in an earlier position.

Then another two points are added for each musical sound. The vowels *a* as in art, *u* as in tuneful or beauty, and *o* as in ode, are considered the musical sounds. Thus the line from Little Boy Blue is given an additional eight points on this score.

From this value is subtracted two points for each alliterative sound in excess of what is pleasing. Another deduction is made for an excess of consonants. There should be not more than two consonant sounds for each vowel sound; otherwise the line will sound harsh. Excess consonants are penalized at the rate of two points each.

When the remaining score is determined, it is then divided by another factor. This last value is obtained by counting all the sounds in the line and adding on extra points in case any two sounds come together which do not flow along smoothly, as when *b* is followed by *p* or *d* by *t*.



MEASURING THEIR BEAUTY

Some geometric forms illustrating the scale of values based on mathematics. The square is rated as the straight-line form having the highest esthetic appeal.

The greater the score for the pleasing, poetical qualities of alliteration, rhyme, and musical sound, in proportion to the total number of sounds in the line, the greater will be the resulting quotient which represents the aesthetic value.

Do you remember this poem by Tennyson?

"Come into the garden, Maud,
For the black bat, Night, has flown,
Come into the garden, Maud,
I am here by the gate alone,
And the Woodbine spices are wafted abroad,
And the musk of the roses blown."

That old favorite has an aesthetic value of .77, next to the highest of those measured by Dr. Birkhoff.

That children have good taste in poetry, too, is indicated by a value of .65 assigned to the nursery rhyme dear to childhood.

"Little boy blue, come blow your horn,
The sheep's in the meadow, the cow's in the corn."

The well-known hymn, "Onward Christian Soldiers," has a value of .51, and the following verse of Amy Lowell received a rating of .64.

"The white mares of the moon rush along the sky
Beating their golden hoofs upon the glass Heavens;
The white mares of the moon are all standing on their hind legs
Pawing at the green porcelain doors of the remote Heavens.

Familiarity with the mathematical formula may serve the artist as an aid in composition, as well as an objective test of the finished work of art. Dr. Birkhoff has actually tried building experimental poems and musical compositions by deliberately placing in them the elements indicated by the formula, blending them as the chemist might combine elements in synthesizing a new substance.

"This experiment was undertaken in order to clarify my own ideas about the nature of poetic composition and to subject them to a test," Dr. Birkhoff said.

"According to the theory it was first of all necessary to start from an idea having some poetic quality."

Here is the idea chosen by Dr. Birkhoff for his experiment in applying a mathematical formula to the writing of verse:

"We may compare if we will, our bits of knowledge to luminous threads which we wind into a compact, luminescent ball. By skillful arrangement of the threads there begins to appear in the center of this ball a bright vision of concepts and laws. If now we add further irrelevant threads, the vision is obscured; and if we unwind the threads in an effort to approach the vision more intimately it becomes more and more faint, and finally disappears."

Making a Poem

"My first attempts to incorporate this idea in poetical form were very unsuccessful," Dr. Birkhoff confessed. "The chief reason for the initial lack of success seems to me now to lie in the fact that the appreciation of the idea was not sufficiently terse. The requirement of terseness is of course fundamental.

"Then one day came without apparent effort the following:

VISION

"Wind and wind the wisps of fire,
Bits of knowledge, heart's desire;
Soon within the central ball
Fiery vision will enthrall.

Wind too long or strip the sphere,
See the vision disappear!"

The aesthetic value of this short poem is .62. Read it aloud and see whether it doesn't appeal to you as musical.

In similar manner has Dr. Birkhoff applied a formula to the composition of original melodies. Perhaps if "Tin

Pan Alley" learns to make use of mathematics in the grinding out of popular ballads, the pleasing qualities, as well as the originiality, of the song of the moment may be increased.

"From the formal point of view, Western music stands pre-eminent by virtue of its purity and its extraordinary degree of development. In poetry there are formal elements which can be isolated and analyzed. But in poetry the meaning is of such dominant importance and so completely eludes formal analysis, that the field of poetry is not pure in the same sense. Similarly it is obvious that other aesthetic fields are inferior to music, either in purity, or else in degree of development as in the case of polygonal form. For, in the case of music, we have a succession of musical sounds, characterized by pitch and time, replete with relationships and devoid of obvious connotations. Furthermore this music has a deep and almost universal appeal."

Paintings, sculpture, and architecture, also have their own order in arrangement—order not only of design but of color as well. Dr. Birkhoff says:

"The 'complexity' of paintings is usually so considerable that they are analogous to ornamental patterns whose constituent ornaments must be appreciated one by one. However, it is decidedly interesting to remark in this connection how a fine composition is always arranged so as to be easily comprehensible."

To illustrate what he meant Dr. Birkhoff showed how, in any work of art, imaginary lines can be drawn across from point to point, following the principal lines of the composition. You will then see that the composition is made up of more or less well defined and entirely comprehensible geometric forms. The light and dark shades are also applied in such a way that they follow a certain order or plan.

Center of Interest

"There should be a natural primary center of interest in the painting and also suitable secondary centers," Dr. Birkhoff explained. "Such a primary center of interest is often taken in the central vertical line of the painting or at least near to it. The elements of order are of course taken to be the same as in the three-dimensional object represented. Finally there are the connotative elements which play a decisive part; a good painting requires a suitable subject just as much as a poem requires a poetical idea.

"The color spectrum, upon which the elements of order involving color necessarily depend, is not without interesting analogy to the gamut of musical tones. The simpler the palette is, the less will be the complexity, so that the palette should be as restricted as the subject permits. Evidently the eye appreciates the repetition of a color, a graded sequence of colors, and a balance of colors or of light and dark values about the centers of interest. All these elements of order are of definite aesthetic importance."

Science News Letter, March 17, 1934

A German scientist succeeded in baking a loaf of bread like ancient Egyptian specimens which had deep hollows in the center, the hollows being presumably used as bowls to hold food eaten in the bread.

ARCHAEOLOGY

Wheat Found in Clay From New Stone Age Hut

FRAGMENTS of clay that once went into the building of a New Stone Age hut, accidentally baked into a brick-like consistency by a chance fire, have preserved for thousands of years evidence that the neolithic farmers grew a species of wheat similar to that cultivated in the earliest fields of Egypt and Mesopotamia. Prof. Fritz Netolitzky, a Rumanian scientist, identified the plant remains after soaking the clay fragments in water and patiently picking them to pieces.

Prof. Netolitzky tells of his discovery in a report to the German scientific journal, *Forschungen und Fortschritte*. The investigation was carried on at the Wallraf-Richartz Museum, Cologne.

Science News Letter, March 17, 1934

PSYCHOLOGY

Repeat Again and Again the Name You Want to Remember

DO YOU easily forget the name of a person after you are introduced to him? The secret of avoiding this embarrassing fault lies in "overlearning" the name by frequent repetitions, Dr. Samuel W. Fernberger, professor of psychology at the University of Pennsylvania, said in a radio address.

"Many years ago I knew a gentleman who was noted for his ability to remember names—a characteristic which was of great value in his life as a politician," Dr. Fernberger said. "He quite frankly told me how he did it. It turned out that he did not have an exceptionally good memory at all. But he frankly and quite consciously set out to overlearn the name of anyone whom he might meet.

"When introduced to Mr. Smith, he did not merely say, 'I am glad to meet you,' but he would say, 'I am glad to meet you, Mr. Smith.' And then he would say, 'And now, Mr. Smith, what did you come to see me about?' 'No, I do not believe that I can help you there, Mr. Smith.' And so on, so that, within a few minutes conversation he had repeated the name ten or a dozen times. It was this repetition—this overlearning—which enabled him to remember the name, not the fact that he possessed an exceptional memory.

"One reason why there is so much forgotten is because you have not sufficiently overlearned what you want to remember. How frequently does the school child say, when he has once succeeded in spelling a word correctly, 'Well, I have that,' and then turns to something else. And if the school child stops at this point, one may expect that 40 per cent. will be forgotten by the end of 20 minutes and 75 per cent. forgotten at the end of a week."

Forming a great many interesting associations between the new idea or the new person and other ideas is another great aid to memory, Dr. Fernberger indicated.

"Be you young or be you old, if you want to remember something, repeat it over and over again, adding as many different associated ideas as you can even after you are sure that you know it," Dr. Fernberger concluded. "In this way you will probably not improve your memory but you will certainly be able to retain and to recall things ever so much better, which after all, is what is important in the practical situation."

Science News Letter, March 17, 1934