Proven path for limiting shortest shortcut

Agony Airlines regularly flies passengers between Salt Lake City and Phoenix and between Phoenix and Los Angeles. The two routes connecting the three cities total 877 miles. However, by adding a hub at Las Vegas and routing all its aircraft through that city, the airline can cut the total length of its routes, now linking four cities, to 859 miles.

This hypothetical example, with its somewhat surprising result, embodies a longstanding mathematical strategy for shortening a network of lines connecting a given set of points: Adding extra points and establishing new links often reduces a network's total length. But what's the most that anyone can save by adding extra points?

Two mathematicians have now proved that the saving can be no better than about 13 percent. Frank K. Hwang of AT&T Bell Laboratories in Murray Hill, N.J., and Dingzhu Du of Academia Sinica in Beijing, China, outline their proof in the December PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES. (Vol. 87, No. 23).

Suppose you want to find the shortest path connecting the three points at the vertices of an equilateral triangle. By positioning a new point in the middle of the triangle, then connecting that point to each vertex, you can find a path that is precisely $\sqrt{3}/2$ times the length obtained simply by joining the points directly – a saving of roughly 13 percent (see diagram).

Indeed, adding extra points in just the right locations can shorten a variety of networks. Designers of long-distance telephone networks, mail-delivery routes and computer circuits often use such a strategy to help minimize the distance something must travel. But until now, network designers really didn't know exactly how much they could save by applying this particular technique.

In 1968, Henry O. Pollak and Edgar N. Gilbert of Bell Labs proposed that the path reduction possible for points arranged in an equilateral triangle was the best anyone could do for any given array of points. They conjectured that no other arrangement of points would ever produce a greater saving.

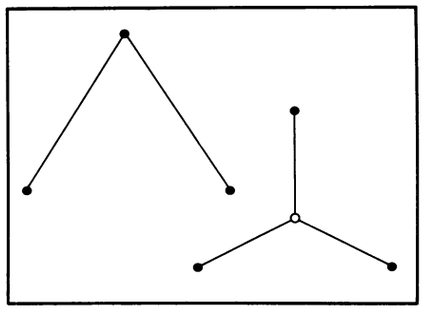
Subsequent work by a number of different mathematicians verified this conjecture for networks encompassing four, five and six points. But a proof of the general case for an arbitrary number of points remained elusive.

"Bell Labs has been interested in this problem for a long, long time," Hwang says. "When I joined Bell Labs, I naturally learned about the problem, and I worked on it [and analogous conjectures] for 15 years."

To crack the problem, Du and Hwang initially tried to generalize previous approaches but all of them involved excessive amounts of computation. They eventually rejected such methods. "We thought that even if we got a proof in that manner, no one would be able read it because it would be so long," Hwang says. For instance, the known proof for the six-point case alone takes 20 printed pages.

Hwang and Du worked out an alternative approach that involves virtually no computation and relies instead on mathematical concepts, which convert the problem from one of enumerating arrangements to one of evaluating certain mathematical expressions. At the scheme's heart lies a "minimax" problem – the type of problem that arises in a game in which one player tries to keep an opponent's payoff to a minimum while maximizing his or her own return.

The resulting proof establishes that no arrangement of points produces a shorter path than that found for an equilateral



triangle of points. There's no way to "slide" the points around to get a greater path reduction.

"Now people can [add extra points to their networks] and have full confidence that all you can save at most is about 13 percent," Hwang says.

Hwang and Du are now studying whether their new approach is useful for solving network problems in three dimensions. They would also like to apply their technique to other mathematical optimization problems. *– I. Peterson*

Women and kids join the cholesterol fray

A regimen of cholesterol-lowering drugs shrinks fatty plaques narrowing the coronary arteries in women – as well as men – who suffer from a genetic defect that puts them at high risk of heart attacks, a new study shows. In a field clogged with studies involving men, this report also gives the first close look at how women respond to such drugs, says Basil Rifkind of the National Heart, Lung, and Blood Institute in Bethesda, Md.

Not all children with high blood cholesterol will grow up to have dangerous levels of cholesterol as adults, a second study finds. Both studies appear in the Dec. 19 JOURNAL OF THE AMERICAN MEDICAL ASSOCIATION.

In the first trial, lasting 26 months, endocrinologist John P. Kane and his colleagues at the University of California, San Francisco, studied 41 women and 31 men with familial hypercholesterolemia, an inherited disorder that leads to dangerously high levels of low-density lipoproteins (LDL), cholesterol-carrying molecules that deposit fatty debris on artery walls. At the study's start, all volunteers exhibited a narrowing from plaque in their coronary arteries, the vessels supplying heart muscle with blood.

The California team advised the entire group to eat a diet low in cholesterol and saturated fats. Roughly half the men and women received aggressive drug therapy to lower their blood cholesterol, the rest mild or no treatment with cholesterol-lowering drugs. The researchers compared before and after X-rays of the volunteers' coronary arteries, obtained by angiography. Their computer analysis showed an average 1.53 percent decrease in vessel obstruction among individuals receiving aggressive drug therapy, compared to an average 0.8 percent rise in the amount of plaque choking coronary arteries in the largely untreated group.

When the California team specifically analyzed data on the women, it confirmed that artery obstruction progressed in women receiving largely dietary guidance, and diminished in women receiving aggressive drug treatment.

Such results may blunt recent criticism regarding the male bias in cardiovascular research. However, a second report by Ronald M. Lauer and William R. Clarke of the University of Iowa in Iowa City suggests the fray over childhood blood cholesterol levels will continue.

The pair analyzed blood cholesterol data collected in the 1970s from 2,367 children and teenagers attending school in Muscatine, Iowa. More than a decade later, the researchers administered follow-up cholesterol tests to 1,461 study participants, then aged 20 to 30. The Iowa team discovered that 75 percent of girls and 56 percent of boys who had high blood cholesterol levels – exceeding the 75th percentile – no longer had high values when tested as young adults.

Because cholesterol screens do not reliably identify future at-risk individuals – and hence may unduly frighten some parents – Thomas B. Newman of the University of California, San Francisco, questions the value of subjecting all children to them. But Gerald S. Berenson of the Louisiana State University Medical Center in New Orleans counters that routine cholesterol tests correctly identify enough children at risk of future heart disease to justify their being part of any public health campaign against heart disease. *– K.A. Fackelmann*