The beauty of mathematics

For P. A. M. Dirac a theoretical physicist's work is a lifelong search for mathematical beauty

by Dietrick E. Thomsen

It was Albert Einstein who turned space from being merely the reference framework for physical action into a participant in the action. In the developed Einsteinian theory, gravitational effects are the result of changes in the curvature of space. Einstein thus solves the centuries-old question about how a body could influence another body located at a distance from it by saying that in fact it doesn't: Body A alters the curvature of the space around it, and this alteration acts on body B in such a way that B appears to be attracted to A.

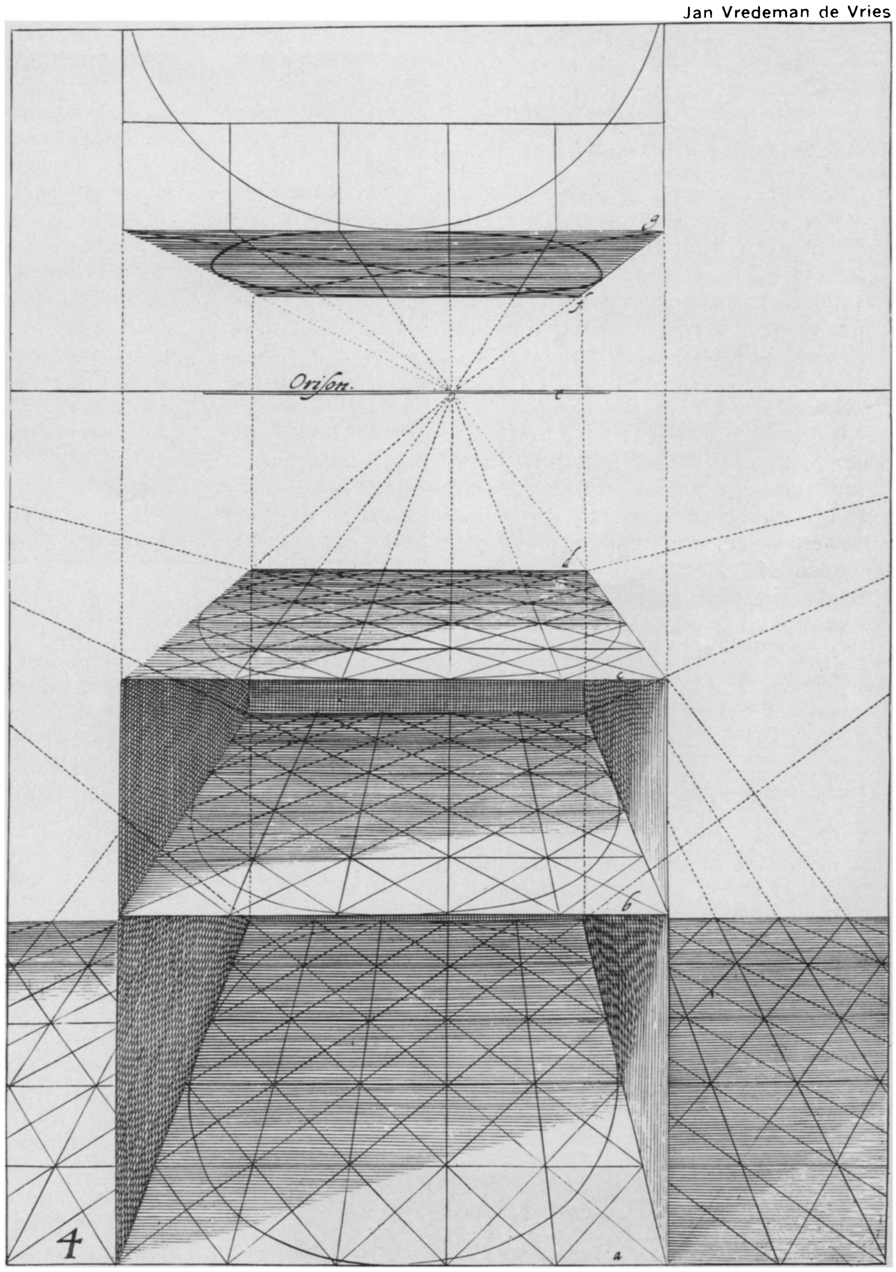
The solution thus makes gravity an effect of geometry. Physicists began to ask: If one of nature's long-range forces is really a geometric effect, why not the other, electromagnetism? Einstein, and many after him, have tried to find a unified field theory that would unite gravity and electromagnetism on a geometrical basis. Recently the thoughts of one of the grand old men of physics, P. A. M. Dirac, who is now at Florida State University at Tallahassee, have turned in this direction.

The work is a further illustration of a principle that Dirac says has guided his work for the last half century: the search for mathematical beauty. In a lecture on how theoretical physicists work that he gave at the recent Coral Gables Conference on Fundamental Interactions, Dirac suggested that a theoretical physicist who works at some remove from the latest experimental work needs a principle that will sustain his conviction in the correctness of his work in what often turns out to be a lengthy wait for experimental confirmation.

Dirac says that he has been attacked for his principle of mathematical beauty on the grounds that beauty is subjective and culturally determined. He is willing to accept such a statement with regard to canons of beauty in such matters as literature and painting, but he insists that just as mathematics can be understood by people of varying cultures so there are standards of mathematical beauty that can be appreciated by people of differing cultural origins.

As an example he cites the Pythagorean relation. The beauty of it is that the square on the hypotenuse turns out to be exactly equal to the sum of the squares on the two other sides. The simplicity and exactness can be widely appreciated, he says. He gives an example in his own life of how the principle led to new physics. About half a century ago Dirac worked out a theory of electrodynamics on the particle level, quantum elec-

Jan Vredeman de Vries



trodynamics. This had in his eyes mathematical beauty, and one of the things that beauty demanded was the existence of antimatter and symmetry between matter and antimatter. The whole idea seemed absurd at the time, but about a decade later Dirac was vindicated by the discovery of the positron, and today antimatter is a basic part of particle physics.

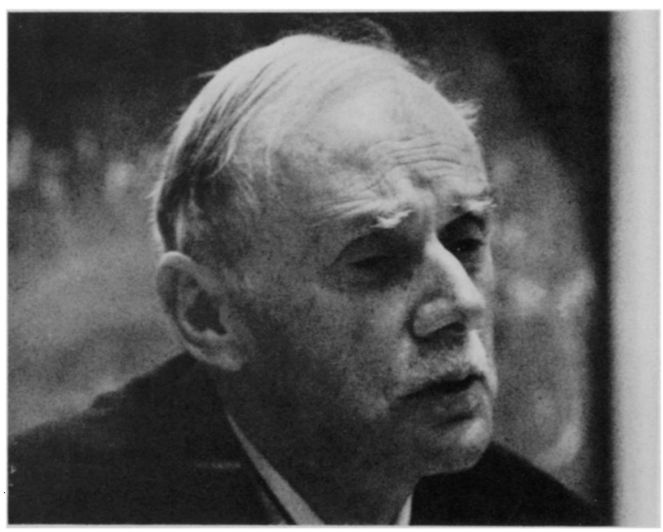
So Dirac takes up the search for a mathematically beautiful theory that will unify gravitation and electromagnetism on a geometric basis. He begins with a 50-year-old solution by Hermann Weyl. Given that gravitation arises from the curvature of space, what feature of geometry might lead to electromagnetic effects? The Weyl answer is that the gauge of the space, its standard for length measurement, could be the factor.

One way to discover the curvature of a space is to move a vector, a quantity such as force that has direction as well as magnitude, around in it. In a curved space a vector that is moved around a closed circuit will arrive back at its starting point pointing in a different direction from what it did at the beginning of the circuit. Likewise, says Dirac following Weyl, you can imagine a space in which the standard for measuring length is not uniform everywhere. If you move a piece of a line around a closed circuit it may return to its starting point a different length. It is from this changing gauge that electromagnetism can be derived.

The reason that Weyl's ideas fell into disfavor is that electromagnetic effects are important in the microcosmic world of atoms and particles, and there this strange kind of space with its ever-changing standard of measurement doesn't seem to fit. For atomic and subatomic measurements nature gives a good and reproducible standard of measurement in the beating of atomic clocks; there seems to be no room for a variable gauge.

Dirac gets around this by postulating two different standards of measurement, an atomic one for the microworld and an Einsteinian one for long-range effects. The Einsteinian one can change according to the ideas of Weyl; the atomic one does not. The ratio between the two of them depends on the age of the universe.

Pursuing these ideas leads to a theory that satisfies Dirac's criteria of mathematical beauty. Among the things the theory predicts is a certain breaking of fundamental physical symmetries. The three basic physical symmetries are those of positive versus negative electric charge, left versus right in space, and going forward versus going backward in time. According to Dirac the two long-range forces are responsible for some breaking of the charge and time symmetries; the breaking of the left-right symmetry (parity) has to be the work of the short-range forces that come into play on the microscopic level. All this seems more or less in accord with current experiment, which notes some rare instances of symmetry breaking.



Dirac: The basic equations should be beautiful.

Univ. of Miami

When Dirac pursues the gravity theory that comes from this attempt at unified field theory, he comes to a result that is not completely Einsteinian. "We've rather gone over to scalar-tensor theory as described by Brans and Dicke," says Dirac. It is a theory in which, unlike the Einsteinian one, the strength of gravity decreases as the universe ages.

According to current experiment the Brans-Dicke theory is in something of a shadow. More and more experiments seem to favor Einstein. But then, 50 years ago, experiment knew nothing of antimatter. It remains to be seen whether in the long run mathematical beauty or the gritty facts of current experiments will win. □